A Robust Version of Convex Integral Functionals

We study the pointwise supremum of convex integral functionals

\[ I_{f, \gamma}(\xi) = \sup_Q \left( \int_\Omega f(\omega, \xi(\omega))Q(d\omega) - \gamma(Q) \right) \]

on \( L^\infty(\Omega, \mathcal{F}, P) \) where \( f : \Omega \times \mathbb{R} \to \mathbb{R} \) is a proper normal convex integrand, \( \gamma \) is a proper convex function on the set of probability measures absolutely continuous w.r.t. \( P \), and the supremum is taken over all such measures. We give a pair of upper and lower bounds for the conjugate of \( I_{f, \gamma} \) as direct sums of a common regular part and respective singular parts; they coincide when \( \text{dom}(\gamma) = \{P\} \) as Rockafellar’s classical result, while both inequalities can generally be strict. We then investigate when the conjugate eliminates the singular measures, which a fortiori yields the equality in bounds, and its relation to other finer regularity properties of the original functional and of the conjugate.

**Keywords**: Convex integral functionals, duality, robust stochastic optimization, financial mathematics.

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