Journal of Convex Analysis 22 (2015) 827–852

K. Owari
Graduate School of Economics, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, 113-0033 Tokyo, Japan
owari@e.u-tokyo.ac.jp

A Robust Version of Convex Integral Functionals

We study the pointwise supremum of convex integral functionals

$$\mathcal{I}_{f,\gamma}(\xi) = \sup_Q \left(\int_\Omega f(\omega, \xi(\omega)) Q(d\omega) - \gamma(Q) \right)$$

on $L^\infty(\Omega, \mathcal{F}, \mathbb{P})$ where $f : \Omega \times \mathbb{R} \to \overline{\mathbb{R}}$ is a proper normal convex integrand, γ is a proper convex function on the set of probability measures absolutely continuous w.r.t. $\mathbb{P}$, and the supremum is taken over all such measures. We give a pair of upper and lower bounds for the conjugate of $\mathcal{I}_{f,\gamma}$ as direct sums of a common regular part and respective singular parts; they coincide when $\mathrm{dom}(\gamma) = \{\mathbb{P}\}$ as Rockafellar's classical result, while both inequalities can generally be strict. We then investigate when the conjugate eliminates the singular measures, which a fortiori yields the equality in bounds, and its relation to other finer regularity properties of the original functional and of the conjugate.

Keywords: Convex integral functionals, duality, robust stochastic optimization, financial mathematics.

MSC: 46N10, 46E30, 49N15, 52A41, 91G80