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revised and extended edition



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PREFACE

Since the appearance of W. Hurewicz and H. Wallman's book "Dimension Theory" in 1941 there have been remarkable developments in dimension theory, especially in the theory for general metric spaces. Though the purpose of this book is to give a rather brief account of modern dimension theory as it has been developed since 1941, the principal results of the classical theory for separable metric spaces are also included. Presupposing only some elementary mathematical knowledge, especially of topology, we shall begin with a brief description of some necessary results in general topology, emphasizing the modern development. No knowledge of dimension theory is assumed, so that the beginning student will be able to read the book without difficulty. However, to the reader who only wishes to get a quick view of the theory, we recommend Chapters I-IV. The author wishes to express his warmest thanks to Prof. J. de Groot who suggested the writing of this book on this interesting theory and helped him in all respects, to Dr. and Mrs. H. de Vries who carefully read the manuscript, gave suggestions and corrected it, especially in its English expression, and to Profs. M. Atsuji, K. Nagami, Y. Kodama and Prof. H. Tamano who helped him in various respects. Without their valuable assistance, this book would never have been written.

Osaka
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Jun-iti Nagata

PREFACE TO THE REVISED EDITION

This is a revised edition of Modern Dimension Theory published in 1964, from North Holland Publishing Co. Since the original edition appeared seventeen years ago, there have been remarkable developments in dimension theory, especially on non-metrizable spaces. Thus a large scale of revisions and additions was made on the original text. For examples, Chapter VII (Dimension of non-metrizable spaces) was wholly rewritten, and Chapter VI (Infinite-dimensional spaces) was greatly innovated by adding two new sections. A new section on Pontrjagin-Schnirelmann's theorem was added to Chapter IV, and sections IV-7 (dimension and ring) and V-3 (dimension and metric function) were also rewritten. Besides, many smaller revisions and additions both in the contents and descriptions would be found in the new edition.

The author wishes to conclude this preface with his heartfelt thanks to the publisher, Heldermann Verlag and to those who pointed out errors in the original edition and gave him various comments and suggestions.

May, 1981
Amsterdam

Jun-iti Nagata