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Horst Herrlich
George E. Strecker

Category Theory

3rd edition

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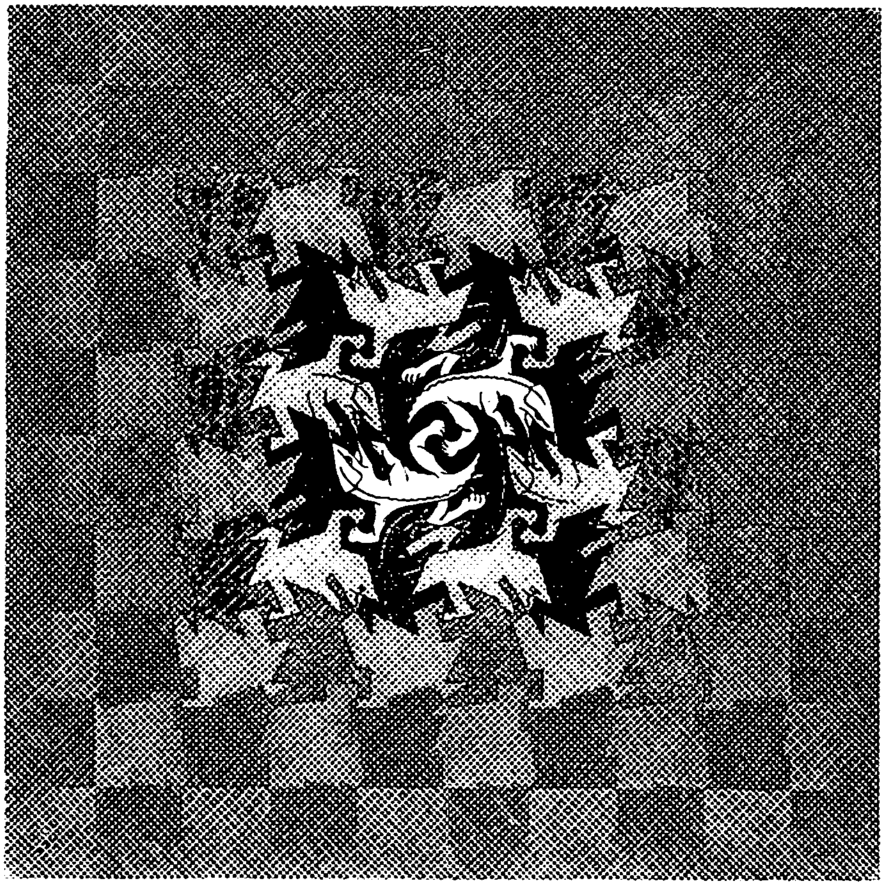
to the Oklawaha and the Ichetucknee



M. C. Escher Ontwikkeling I (Escher Foundation—Collection Haags Gemeentemuseum—The Hague)

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Preface

Our purpose in writing this book is to present the theory of categories at the earliest moment at which the reader can appreciate it, that is, as soon as he becomes reasonably acquainted with set theory, modern algebra, and general topology. It is hoped that such a presentation will help him to prepare more adequately for advanced topics in these subjects and in algebraic topology. Thus the book is designed for normal use during the early stages of graduate study—or possibly in honors courses for undergraduates. However, this does not preclude its use as "armchair reading" for mature mathematicians who have not yet had formal exposure to the subject.

The attempt is made to present category theory mainly as a convenient new language—one which ties together earlier notions, which puts many existing results in their proper perspective, and which provides a means for appreciation of the unity that exists in modern mathematics, despite the increasing tendencies toward fragmentation and specialization.

Our approach is heavily dependent upon numerous examples and exercises drawn from set theory, algebra, and topology. By continually tying down new notions to well-known concrete examples, it is hoped that the relatively high level of abstraction that is embodied in category theory can be kept from becoming a high level of obfuscation.

Throughout the book we have striven to achieve a pedagogical soundness that would make it appropriate for use even without the aid of an instructor. Some care has been taken to arrive at a flow of topics that would be "self-motivating" and to resist the temptation to abstract very quickly for the sake of efficiency. For example, an efficient approach would have been to define the more general concepts first, and then to specialize them. We have taken quite the opposite approach, by first abstracting to a categorical context various concrete notions within the reader's probable realm of experience and then, when common features among these notions begin to appear, abstracting them to even more general notions. Thus, for example, we have used the usual products of sets, groups, modules, and topological spaces to motivate categorical products, have investigated several special cases of limits before introducing the general notion, and have delayed introduction of the central and very important notion of adjoint functor until the point where it can be easily appreciated.

The book is divided into eleven chapters that represent natural "clusters" of topics, and it is intended that these be covered sequentially (at least through Chapter VII). If time is short, Chapters VIII and X could be omitted. It should be mentioned that even on first reading, Chapter II (Foundations) should be covered, since in category theory the distinction between sets and classes is quite important. (To ask, for instance, that a category possess products for all *set*-indexed families of its objects is far different from asking the same for all *class*-indexed families.) To facilitate references, each chapter is divided into sections that are numbered sequentially throughout the book and all items within a given section are numbered sequentially throughout it. The symbol □ has been used to designate the ends of proofs as well as to mark those instances where the proof is left to the reader (and is thus an implied exercise). The examples may also be considered to be implied exercises, depending upon one's inclination and mathematical background. The exercises that appear at the end of each section have been designed both as an aid in the understanding of the material of the section as well as a means to begin to apply it elsewhere. They range widely in their difficulty.

Category theory is a relatively young field without settled terminology and notation. For this reason we have in general tried to use terminology that has become "standard" over the last several years, and have strayed from this principle only when we felt that there was a compelling reason to do so. For example, we have used the term "dense functor" where the more standard term is "representative functor", both because "dense" seems to be more descriptive and because the confusion between *representative* and *representable* functors is avoided. In the index we have tried to include the standard terms so that when the book is used for reference, confusion should be unlikely. In this connection we confess to having introduced the new terms of "source" and "sink". However, we believe that these notions are fundamental and quite often lead to useful descriptions and visualizations. At this point an apology is also needed regarding the "order" used when denoting the composition of morphisms. Certainly mankind (or at least mathematiciankind) will long be plagued by the regrettable historical accident that the value of a function f at a point x has been denoted $f(x)$ rather than $(x)f$. Because of this notation, the value of $\bullet \xrightarrow{f} \bullet \xrightarrow{g} \bullet$ at x is written $g(f(x))$ and, consequently, the composition of f and g is denoted by $g \circ f$. This form of designation constitutes a switch in the order, and it would be far preferable both aesthetically and practically to write the composition as $f \circ g$. However, we have chosen to adopt the more traditional notation simply because its use by mathematicians is practically universal and because a totally consistent change of notation would have certainly led to undue confusion and alienation—[a sequence, for example, would have to be denoted by $({}_nx)$ rather than (x_n), and the nth homology group of a space X by $(X)_nH$ rather than $H_n(X)$].

Throughout the text we have included very few references to the literature and have made no attempt to provide a historical development of the theory. There are several reasons for this. First of all, category theory is too young to have a real "history". Secondly our treatment of the subject is intended to be only on an introductory level. Also, many categorical results have been found independently (often in different forms) by several individuals. Finally, many of the results are actually (as far as their "essence" is concerned) older than category theory itself. For those who wish to continue the study of categories or to do research in the area, we have provided a fairly extensive bibliography.

H.H.
G.E.S.

Preface to the Second Edition

Since the first edition of *Category Theory* appeared in 1973, we have been gratified by the interest that it has generated among both students and professional mathematicians who had not previously had detailed exposure to the subject. The field of category theory has grown in many new directions since then. Yet we believe there is still a need for an introductory level text that relies heavily on motivation via numerous examples and exercises.

Thus we are indeed pleased by the opportunity provided by Heldermann Verlag to present a second edition that incorporates many improvements and corrections that we have wished to make. In this connection it should be noted that a list of some errata of a typographical nature is included on page 382.

We wish especially to thank those who have contributed suggestions for revisions and to express our appreciation to Dr. Norbert Heldermann for his many efforts in making this edition a success.

H.H.
G.E.S.

Preface to the Third Edition

The second edition of this text has been out of print for several years. However, due to the fact that the categorical language has become indispensable in many areas of mathematics as well as in the rapidly growing field of theoretical computer science, there has been an undiminished demand for an elementary introduction to category theory. Therefore we are pleased that our publisher Prof. Dr. Norbert Heldermann has made this work available again. Further information is available from the home page of this book, which you find at the web-site of the publisher at www.heldermann.de.

Heldermann Verlag has also made available an electronic edition of our more advanced text:

Abstract and Concrete Categories. The Joy of Cats,

written jointly with Jiri Adamek. It is accessible via:
http://www.heldermann.de/Ebooks/ebook3.htm

Bremen 2007 — H.H.
Manhattan 2007 — G.E.S.

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We also owe special thanks to the late Johannes de Groot for his keen interest in our work and for his inspiring discussions.

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Finally, we would like to extend our general thanks to all categorists (both latent and declared) whose results we have used so freely.