

PREFACE

Some mathematical concepts appear to be “unavoidable”, e. g. that of natural numbers. For other concepts such a claim seems debatable, e. g., for the concepts of real numbers or of groups. Other concepts — within certain limits — seem to be quite arbitrary, their use being based more on historical accidents than on structural necessities. A good example is the concept of topological spaces: compare such “competing” concepts as metric spaces, convergence spaces, pseudotopological spaces, uniform spaces, nearness spaces, frames respectively locales, etc. What are the structural “necessities” or at least “desirabilities”? Category theory provides a language to formulate such questions with the kind of precision needed to analyse advantages and disadvantages of various alternatives. In particular, category theory enables us to decide whether certain mathematical “disharmonies” are due to inherent structural features or rather to chance occurrences, and in the latter case helps to “set things right”. This, naturally, is not the only use of category theory. Category theory also allows us to detect structural similarities between mathematical phenomena, which at first glance may appear to be completely unrelated, and thus enables us to formulate concepts of great generality (e. g., the concept of limits) and to prove results with applications in diverse areas of mathematics (e. g., the adjoint functor theorems). Category theory helps us to discover that in various proofs certain parts are nothing more than “standard” categorical arguments; thus it helps us to separate the “trivial” from the “deep”. And, and, and,

The purpose of this book is to serve as a (direly needed) text for a seminar in applied category theory. The treatise contains 23 articles, which are intended to paradigmatically illustrate the above claims, i. e., to demonstrate how categorical thinking contributes to a better understanding of certain mathematical phenomena by putting them into particularly suitable contexts; in other words: to let the reader experience and appreciate

CATEGORY THEORY AT WORK

All articles are especially written for this project, have been discussed among the authors at a workshop held at the University of Bremen July 16 and 17, 1990, and have undergone a severe refereeing process. While several articles contain known results in a new and hopefully particularly attractive presentation, other articles contain ideas and results, presented here for the first time and thus offer new results of interest not only to beginners but to “seasoned” categorists as well.

Although the editors attempted to reach a certain uniformity in presentation, it cannot surprise anybody who knows mathematicians that our efforts where only partially successful. Thus, while most articles are supplied with complete proofs, a few are more of a survey nature, providing detailed references instead of proofs (and thus may serve as useful sources for some MSc-theses). However, all authors have seriously tried to write in such a way that students with some elementary background in category theory can follow the presentation without undue effort. The articles have been arranged by topic; however, they can be studied in an arbitrary order.

For the editors it is a pleasure to express our thanks to all those, who helped to make this project a success, first of all to the authors for their manuscripts, to the referees for doing their unrewarding and unsung job, to the participants of the Bremen workshop, to the University of Bremen for financial support of the workshop, to the mathematician-artist Marcel Ern  for his art work, to our secretary Mrs. Birgit Feddersen for putting the manuscript in proper shape, and to the publisher for his interest in this project.

Bremen, March 1991

The Editors