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Existence and Uniqueness of Common Solutions of Strict Stampacchia and Minty Variational Inequalities with Non-Monotone Operators in Banach Spaces

We study the existence of common solutions of the Stampacchia and Minty variational inequalities associated to non-monotone operators in Banach spaces, as a consequence of a general saddle-point theorem. We prove, in particular, that if $(X, \|\cdot\|)$ is a Banach space, whose norm has suitable convexity and differentiability properties, $B_\rho := \{x \in X : \|x\| \leq \rho\}$, and $\Phi : B_\rho \to X^*$ is a C^1 function with Lipschitzian derivative, with $\Phi(0) \neq 0$, then for each $r > 0$ small enough, there exists a unique $x^* \in B_r$, with $\|x\| = r$, such that $\max\{\langle \Phi(x^*), x^*-x \rangle, \langle \Phi(x), x^*-x \rangle\} < 0$ for all $x \in B_r \setminus \{x^*\}$. Our results extend to the setting of Banach spaces some results previously obtained by B. Ricceri in the setting of Hilbert spaces.

Keywords: Saddle point, minimax theorem, Banach space, modulus of convexity, C^1 function, Stampacchia and Minty variational inequalities, ball, non-monotone operators.

MSC: 47J20, 49J35, 49J40.