

L. Kong

Dept. of Mathematics, University of Tennessee, Chattanooga, U.S.A.
lingju-kong@utc.edu

Multiplicity Theorems for Biharmonic Kirchhoff-Type Elliptic Problems

We study the existence of multiple weak solutions for the biharmonic Kirchhoff-type elliptic problem

$$\begin{cases} M \left(\int_{\Omega} (|\Delta u|^p + |\nabla u|^p) dx \right) (\Delta_p^2 u - \Delta_p u) = \sum_{i=1}^k \alpha_i(x) f_i(u) + \gamma(x) \text{ in } \Omega, \\ u = \Delta u = 0 \text{ on } \partial\Omega, \quad \int_{\Omega} (|\Delta u|^p + |\nabla u|^p) dx < \rho. \end{cases}$$

We establish necessary and sufficient conditions on $f_i, i = 1, \dots, k$, under which there exists functions $\alpha_i, \gamma \in C(\bar{\Omega}), i = 1, \dots, k$, such that the above problem has at least two weak solutions. Our proof uses the variational approaches and relies on an existence result for critical points of functionals in Banach spaces recently obtained by Ricceri.

Keywords: Kirchhoff-type problems, p -Laplacian operator, p -biharmonic operator, weak solutions, critical points, contraction mapping theorem.

MSC: 35G30, 35J58, 49J35.