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Nonlinear Semigroup Approach to the Hamilton-Jacobi Equation – a Toy Model

We discuss the existence and multiplicity problem of viscosity solution to the Hamilton-Jacobi equation

$$h(x, d_x u) + \lambda(x)u = c, \quad x \in M,$$

where M is a closed manifold and $\lambda : M \rightarrow \mathbb{R}$ changes signs on M , via nonlinear semigroup method. It turns out that a bifurcation phenomenon occurs when the parameter c strides over some critical value. As an application of the main result, we analyse the structure of the set of viscosity solutions of an one-dimensional example in detail.

Keywords: Hamilton-Jacobi equations, viscosity solutions, bifurcation phenomenon.

MSC: 35F21,35D40,35A02,35B32.