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Nonlinear Curl-Curl Problems in \mathbb{R}^3

We survey recent results concerning ground states and bound states $u \colon \mathbb{R}^3 \to \mathbb{R}^3$ to the curl-curl problem

$$\nabla \times (\nabla \times u) + V(x)u = f(x, u)$$
 in \mathbb{R}^3 ,

which originates from the nonlinear Maxwell equations. The energy functional associated with this problem is strongly indefinite due to the infinite dimensional kernel of $\nabla \times (\nabla \times \cdot)$. The growth of the nonlinearity f is superlinear and subcritical at infinity or purely critical and we demonstrate a variational approach to the problem involving the generalized Nehari manifold. We also present some refinements of known results.

Keywords: Time-harmonic Maxwell equations, ground state, variational methods, strongly indefinite functional, curl-curl problem, Orlicz spaces, N-functions.

MSC: 35Q60; 35J20, 78A25.