Minimax Theory and its Applications 07 (2022) 303–320

M. Liu
Dept. of Mathematical Sciences, Tsinghua University, Beijing, P. R. China
liumq20@mails.tsinghua.edu.cn

W. Zou
Dept. of Mathematical Sciences, Tsinghua University, Beijing, P. R. China
zou-wm@mail.tsinghua.edu.cn

Normalized Solutions for a System of Fractional Schrödinger Equations with Linear Coupling

We study the normalized solutions of the following fractional Schrödinger system:

$$\begin{cases} (-\Delta)^s u = \lambda_1 u + \mu_1 |u|^{p-2}u + \beta v & \text{in } \mathbb{R}^N, \\ (-\Delta)^s v = \lambda_2 v + \mu_2 |v|^{q-2}v + \beta u & \text{in } \mathbb{R}^N, \end{cases}$$

with prescribed mass $\int_{\mathbb{R}^N} u^2 = a$ and $\int_{\mathbb{R}^N} v^2 = b$, where $s \in (0,1)$, $2 < p, q \le 2_s^*$, $\beta \in \mathbb{R}$ and μ_1, μ_2, a, b are all positive constants. Under different assumptions on p, q and $\beta \in \mathbb{R}$, we succeed to prove several existence and nonexistence results about the normalized solutions. Specifically, in the case of mass-subcritical nonlinear terms, we overcome the lack of compactness by establishing the least energy inequality and obtain the existence of the normalized solutions for any given $a, b > 0$ and $\beta \in \mathbb{R}$. While for the mass-supercritical case, we use the generalized Pohozaev equality to get the boundedness of the Palais-Smale sequence and obtain the positive normalized solution for any $\beta > 0$. Finally, in the fractional Sobolev critical case i.e., $p = q = 2_s^*$, we give a result about the nonexistence of the positive solution.

Keywords: Fractional Laplacian, Schroedinger system, normalized solutions.

MSC: 35R11, 35B09, 35B33.