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Nonrelativistic Limit of Ground State Solutions for Nonlinear Dirac-Klein-Gordon Systems

We study the nonrelativistic limit and some properties of the solutions

$$(\psi, \phi) := (u, v, \phi) \in \mathbb{C}^2 \times \mathbb{C}^2 \times \mathbb{R}$$

for the following nonlinear Dirac-Klein-Gordon systems:

$$\begin{cases} ic\sum_{k=1}^{3} \alpha_k \partial_k \psi - mc^2 \beta\psi - \omega\psi - \lambda\phi\beta\psi = |\psi|^{p-2}\psi, \\ \\ -\Delta\phi + c^2 M^2 \phi = 4\pi\lambda(\beta\psi)\cdot\psi, \end{cases}$$

where $p \in [\frac{12}{5}, \frac{8}{3}]$, c denotes the speed of light, $m > 0$ is the mass of the electron. We show that the first component u and the last one ϕ of ground state solutions for nonlinear Dirac-Klein-Gordon systems converge to zero and the second one v converges to corresponding solutions of a coupled system of nonlinear Schrödinger equations as the speed of light tends to infinity for electrons with small mass. Moreover, we also prove the uniform boundedness and the exponential decay properties of the solutions for the nonlinear Dirac-Klein-Gordon systems with respect to the speed of light c.

Keywords: Nonlinear Dirac-Klein-Gordon systems, nonrelativistic limit, ground state solution.

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