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Existence and Local Uniqueness of Normalized Multi-Peak Solutions to a Class of Kirchhoff Type Equations

We study the existence and local uniqueness of multi-peak solutions to the following Kirchhoff type equations

$$-\Big(a + b_\lambda \int_{\mathbb{R}^3} |\nabla u_\lambda|^2\Big)\Delta u_\lambda + \big(\lambda + V(x)\big)u_\lambda = \beta_\lambda u_\lambda^p,$$

where $u_\lambda \in H^1(\mathbb{R}^3)$, $u_\lambda > 0$ in $\mathbb{R}^3$, with normalized L^2-constraint, that is,

$$\int_{\mathbb{R}^3} u_\lambda^2 = 1,$$

where $a > 0$, $p \in (1,5)$ are constants, $\lambda, b_\lambda, \beta_\lambda > 0$ are parameters, and $V(x)\colon \mathbb{R}^3 \to \mathbb{R}^1$ is a bounded continuous function. Physicists are very interested in normalized solutions. Compared to finding multi-pick solutions to the equation without normalized L^2-constraint one is facing here some new difficulties in getting normalized solutions to the equation. We first prove that for the case of $3 < p < 5$, there exist sequences $\{b_\lambda\}_\lambda$ and $\{\beta_\lambda\}_\lambda$ such that for any sufficiently large $\lambda > 0$, one can construct multi-peak solutions u_λ of some given form to the above equation by using the Lyapunov-Schmidt reduction method under some mild assumptions on the function $V(x)$. In the proof of the above existence result, we consider the three cases of $p = 11/3$, $3 < p < 11/3$ and $11/3 < p < 5$ separately, which correspond to the cases of mass critical, subcritical and supercritical in physics respectively. Then, applying the blow-up technique and the local Pohozaev identities we obtain a uniqueness result of multi-peak solutions for the case of $3 < p < 5$. The difficulties caused by the nonlocal term and normalized L^2-constraint are overcome.

Keywords: Kirchhoff type equations, multi-peak normalized solutions, Lyapunov-Schmidt reduction, local Pohozaev identity, existence and local uniqueness.

MSC: 35J20, 35J60, 35J92.