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Multiplicity of Solutions on a Nehari Set in an Invariant Cone

For $1 < p < 2$ and q large, we prove the existence of two positive, nonconstant, radial and radially nondecreasing solutions of the supercritical equation

$$-\Delta_p u + u^{p-1} = u^{q-1}$$

under Neumann boundary conditions, in the unit ball of $\mathbb{R}^N$. We use a variational approach in an invariant cone. We distinguish the two solutions upon their energy: one is a ground state inside a Nehari-type subset of the cone, the other is obtained via a mountain pass argument inside the Nehari set.

As a byproduct of our proofs, we detect the limit profile of the low energy solution as $q \to \infty$ and show that the constant solution 1 is a local minimizer on the Nehari set. This marks a strong difference with the case $p \geq 2$.

Keywords: Quasilinear elliptic equations, Sobolev-supercritical nonlinearities, Neumann boundary conditions, Radial solutions.

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