Minimax Theory and its Applications 06 (2021) 145–154

Y. Kim
Department of Mathematics, University of Wisconsin, Madison, WI 53706, U.S.A.
yeonkim@math.wisc.edu

On the Uniqueness of Solutions to One-Dimensional Constrained Hamilton-Jacobi Equations

The goal of this paper is to study the uniqueness of solutions to a constrained Hamilton-Jacobi equation

$$\begin{cases} u_t = u_x^2 + R(x, I(t)) & \text{in } \mathbb{R} \times (0, \infty), \\ \max_{\mathbb{R}} u(\cdot, t) = 0 & \text{on } [0, \infty), \end{cases}$$

with an initial condition $u(x, 0) = u_0(x)$ on $\mathbb{R}$. A reaction term $R(x, I(t))$ is given while $I(t)$ is an unknown constraint (Lagrange multiplier) that forces maximum of u to be always zero. In the paper, we prove uniqueness of a pair of unknowns (u, I) using the dynamic programming principle for a particular class of non-separable reaction $R(x, I(t))$ when the space is one-dimensional.

Keywords: Hamilton-Jacobi equation with constraint, selection-mutation model.

MSC: 35A02, 35F21, 35Q92.