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Hopf Formulas for Nonlinear Obstacle Problems

A Hopf formula is derived for

$$\max\{u_t + H(Du),\, h(t,x) - u\} = 0, \quad u(T,x) = g(x) \geq h(t,x),$$

where g is assumed convex and $x \mapsto h(t,x)$ is also convex. This generalizes a formula without time dependent obstacle due to Subbotin. A Hopf formula for a concave obstacle is also derived. In addition, the Hopf formula for the obstacle problem with quasiconvex g is established. Next we consider the double obstacle problem. Assume the two obstacles $g_1(x) \leq g_2(x)$ are given functions, both convex or both concave. The nonlinear double obstacle variational inequality $\max\{\min\{u_t + H(Du), g_2 - u\}, g_1 - u\} = 0$ on $(-\infty, T) \times \mathbb{R}^n$, with terminal data either g_2 in the convex case and g_1 in the concave case has a viscosity solution given by a Hopf type formula. These formulas are derived by using differential games with stopping times.

Keywords: Differential games, stopping times, Hopf formula, double obstacle.

MSC: 49K35, 49K45, 49L25, 49L20, 90C47.