Minimax Theory and its Applications 04 (2019) 217–229

G. Anello
Dept. of Mathematics and Computer Science, Physical Science and Earth Science, University of Messina, Viale F. Stagno d'Alcontres 31, 8166 Messina, Italy
ganello@unime.it

Existence Results and Strong Maximum Principle for a Resonant Sublinear Elliptic Problem

Let Ω be a bounded smooth connected open set in $\mathbb{R}^N$ and let λ_1 be the first eigenvalue of the Laplacian on Ω. We study the resonant elliptic problem

$$\begin{cases} -\Delta u = \lambda_1 u + u^{s-1} - \mu u^{r-1}, & \text{in} \quad \Omega \\ u \geq 0, & \text{in} \quad \Omega \\ u_{|\partial\Omega} = 0 \end{cases}$$

where $s \in]1,2[$, $r \in]1,s[$, and $\mu \in]0,+\infty[$. An existence result of nonzero solutions is established via minimax and perturbation methods. Furthermore, for μ large enough, we prove a Strong Maximum Principle for the solutions of this problem. In particular, we extend to higher dimension an analogous recent result obtained in the one-dimensional case via the time-mapping method.

Keywords: Sublinear elliptic problem, resonance, nonnegative solution, positive solution, minimax method, mountain pass, strong maximum principle.

MSC: 35J20, 35J25