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A New Minimax Theorem for Linear Operators

The aim of this note is to prove the following minimax theorem which generalizes a result by B. Ricceri and extends a previous result of the author: let E be a infinite-dimensional Banach space, F be a Banach space, X be a convex subset of E whose interior is non-empty for the weak topology on bounded sets, Δ a finite-dimensional convex compact subset of $\mathcal{L}(E,F)$, $\varphi\colon F\to\mathbb{R}$ be a continuous convex coercive map, and $\psi\colon\Delta\to\mathbb{R}$ a convex continuous function. Assume moreover that Δ contains at most one compact operator. Then

$$\sup_{x\in X}\inf_{T\in\Delta}\big(\varphi(Tx)+\psi(T)\big)=\inf_{T\in\Delta}\sup_{x\in X}\big(\varphi(Tx)+\psi(T)\big)\ .$$

Keywords: Minimax, Banach spaces, linear operators.

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