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On a Minimax Problem for Ovals

For a bounded metric space (X, d) we consider the quantity

$$\delta(X) := \inf_{p \in X} \sup_{q \in X} d(p, q).$$

This purely metric invariant is known from approximation theory as the relative Chebyshev radius of X w.r.t. X itself. Despite its obvious meaning, the invariant $\delta(X)$ seems rather untouched in the geometric literature. Here we discuss, for a plane convex curve $X = \Gamma$, an isoperimetric type inequality between $\delta(\Gamma)$ and the perimeter $L(\Gamma)$, namely $L(\Gamma) \geq \pi \cdot \delta(\Gamma)$. Though the most general case is open there are classes of curves where definitive versions of the inequality are possible, including a discussion of equality. For quadrilaterals there is a surprising occurrence of 'magic kites' as possible extremals. A finite algorithm for polygons is established, and numerous experiments with it yield strong support for a general validity of the inequality.

Keywords: Metric invariant, relative Chebyshev radius, isoperimetric inequality.

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