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A Minimax Theorem for Linear Operators

The aim of this note is to prove the following minimax theorem which generalizes a result by B. Ricceri: let E be an infinite-dimensional Banach space not containing ℓ^1, F be a Banach space, X be a convex subset of E whose interior is non-empty for the weak topology on bounded sets, S and T be linear and continuous operators from E to F, $\varphi : F \to \mathbb{R}$ be a continuous convex coercive map, $J \subset \mathbb{R}$ a compact interval and $\psi : J \to \mathbb{R}$ a convex continuous function. Assume moreover that $S \times T$ has a closed range in $F \times F$ and that S is not compact. Then

$$\sup_{x\in X} \inf_{\lambda\in J} \big(\varphi(Tx - \lambda Sx) + \psi(\lambda)\big) = \inf_{\lambda\in J} \sup_{x\in X} \big(\varphi(Tx - \lambda Sx) + \psi(\lambda)\big) .$$

In particular, if φ is the norm of F and $\psi = 0$, we get

$$\sup_{x\in X} \inf_{\lambda\in J} \|Tx - \lambda Sx\| = \inf_{\lambda\in J} \sup_{x\in X} \|Tx - \lambda Sx\| .$$

Keywords: Minimax, Banach spaces, linear operators.

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