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Minimax Theory and its Applications 01 (2016) 065–082

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**Local Regularity for Mean-Field Games in the Whole Space**

We investigate the Sobolev regularity for mean-field games in the whole space  $\mathbb{R}^d$ . This is achieved by combining integrability for the solutions of the Fokker-Planck equation with estimates for the Hamilton-Jacobi equation in Sobolev spaces. To avoid the mathematical challenges due to the lack of compactness, we prove an entropy dissipation estimate for the adjoint variable. This, together with the non-linear adjoint method, yields uniform estimates for solutions of the Hamilton-Jacobi equation in  $W_{loc}^{1,p}(\mathbb{R}^d)$ .