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Local Noncommutative De Leeuw Theorems Beyond Reductive Lie Groups

Let Γ be a discrete subgroup of a unimodular locally compact group G. M. Caspers et al. [*Local and multilinear noncommutative de Leeuw theorems*, Math. Ann. 388 (2024) 4251–4305] showed that the L_p-norm of a Fourier multiplier $m\colon G \to \mathbb{C}$ on Γ can be bounded locally by its L_p-norm on G, modulo a constant $c(A)$ which depends on the support A of $m|_\Gamma$. In the context where G is a connected Lie group with Lie algebra $\mathfrak{g}$, we develop tools to find explicit bounds on $c(A)$. We show that the problem reduces to:

(1) The adjoint representation of the semisimple quotient $\mathfrak{s} = \mathfrak{g}/\mathfrak{r}$ of $\mathfrak{g}$ by the radical $\mathfrak{r} \subseteq \mathfrak{g}$ (which was handled in the paper of M. Caspers et al. cited above).

(2) The action of $\mathfrak{s}$ on a set of real irreducible representations that arise from quotients of the commutator series of $\mathfrak{r}$.

In particular, we show that $c(G) = 1$ for unimodular connected solvable Lie groups.

Keywords: Fourier multipliers, almost invariant neighbourhoods.

MSC: 22E15, 43A15, 43A22, 22D25, 46L51.