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Shimura Operators for Certain Hermitian Symmetric Superpairs

We give a partial super analog of a result obtained by Sahi and Zhang relating Shimura operators and certain interpolation symmetric polynomials. In particular, we study the pair $(\mathfrak{gl}(2p|2q), \mathfrak{gl}(p|q)\oplus\mathfrak{gl}(p|q))$, define the Shimura operators in $\mathfrak{U}(\mathfrak{g})^{\mathfrak{k}}$, and using a new method, prove that their images under the Harish-Chandra homomorphism are proportional to Sergeev and Veselov's Type BC interpolation supersymmetric polynomials under the assumption that a family of irreducible $\mathfrak{g}$-modules are spherical. We prove this conjecture using the notion of quasi-sphericity for Kac modules when $p = q = 1$, and give explicit coordinates of (quasi-)spherical vectors.

Keywords: Shimura operators, symmetric superpairs, Lie superalgebras, interpolation polynomials.

MSC: 17B10, 17B60, 05E10, 81Q60.