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Bounding the Norm of the Derivative of the Lie Exponential Map for Connected Lie Groups

Let G be a real connected Lie group with a left invariant metric d , $\mathfrak{g}$ its Lie algebra, $\exp : \mathfrak{g} \rightarrow G$ be the Lie exponential map, and ad be the adjoint representation of $\mathfrak{g}$. In this paper we use matrix algebra and Jordan normal form to derive a set of upper and lower bounds for $|d\exp_x(y)|$, $x, y \in \mathfrak{g}$ that generally are exponential type functions of the eigenvalues of ad_x . These bounds provide useful information about the exponential map and the way it relates the Euclidean metric of $\mathfrak{g}$ and the left invariant metric of G . For Lie groups for which the exponential map is a diffeomorphism, we investigate conditions under which the exponential map is a quasi-isometry. This is obviously true if G is isomorphic to $\mathbb{R}^n$. We prove that the exponential map is a quasi-isometry only when G is isomorphic to $\mathbb{R}^n$.

Keywords: Lie group, exponential map, adjoint, quasi-isometry.

MSC: 22E15, 22E60.