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Geodesic Completeness of some Lorentzian Simple Lie Groups

We investigate geodesic completeness of left-invariant Lorentzian metrics on a simple Lie group G when there exists a left-invariant Killing vector field Z on G . Among other results, it is proved that if Z is timelike, or G is strongly causal and Z is lightlike, then the metric is complete. The situation is considerably elaborate when Z is spacelike, as our study of the special complex Lie group $SL_2(\mathbb{C})$ illustrates. We show that the existence of a lightlike vector field Z on $SL_2(\mathbb{C})$, implies geodesic completeness. When Z is spacelike and orthogonal to $\sqrt{-1}Z$, we characterize complete metrics on $SL_2(\mathbb{C})$.

Keywords: (Semi)simple Lie group, left-invariant metric, Lorentzian metric, Killing vector field, left-invariant vector field, semisimple element, nilpotent element, compact element, strongly causal, dual Euler equation, generalized conical spiral, limit curve, fir

MSC: 53C22, 53C50, 57M50, 17B08, 22E30.