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Full Projective Oscillator Representations of Special Linear Lie Algebras and Combinatorial Identities

Using the projective oscillator representation of $\mathfrak{sl}(n+1)$ and Shen's mixed product for Witt algebras, Y. Zhao and the second author [*Generalized projective representations for $\mathfrak{sl}(n+1)$* , J. Algebra 328 (2011) 132–154] constructed a new functor from $\mathfrak{sl}(n)$ -**Mod** to $\mathfrak{sl}(n+1)$ -**Mod**. In this paper, we start from $n = 2$ and use the functor successively to obtain a full projective oscillator realization of any finite-dimensional irreducible representation of $\mathfrak{sl}(n+1)$. The representation formulas of all the root vectors of $\mathfrak{sl}(n+1)$ are given in terms of first-order differential operators in $n(n+1)/2$ variables. One can use the result to study tensor decompositions of finite-dimensional simple modules by solving certain first-order linear partial differential equations, and thereby obtain the corresponding physically interested Clebsch-Gordan coefficients and exact solutions of Knizhnik-Zamolodchikov equation in WZW model of conformal field theory.

Keywords: Special linear Lie algebra, projective oscillator representation, simple module, singular vectors, combinatorial identities.

MSC: 17B10; 05A19.