

A. Chirvasitu

Department of Mathematics, University at Buffalo, U.S.A., New York, U.S.A.
achirvas@buffalo.edu

Lie-Algebra Centers via De-Categorification

Let \mathfrak{g} be a Lie algebra over an algebraically closed field \mathbb{K} of characteristic zero. Define the universal grading group $\mathcal{C}(\mathfrak{g})$ as having one generator g_ρ for each irreducible \mathfrak{g} -representation ρ , one relation $g_\pi = g_\rho^{-1}$ whenever π is weakly contained in the dual representation ρ^* (i.e. the kernel of π in the enveloping algebra $U(\mathfrak{g})$ contains that of ρ^*), and one relation $g_\rho = g_{\rho'} g_{\rho''}$ whenever ρ is weakly contained in $\rho' \otimes \rho''$.

The main result is that attaching to an irreducible representation its central character gives an isomorphism between $\mathcal{C}(\mathfrak{g})$ and the dual \mathfrak{z}^* of the center $\mathfrak{z} \leq \mathfrak{g}$ when \mathfrak{g} is (a) finite-dimensional solvable; (b) finite-dimensional semisimple. The group $\mathcal{C}(\mathfrak{g})$ is also trivial when the enveloping algebra $U(\mathfrak{g})$ has a faithful irreducible representation (which happens for instance for various infinite-dimensional algebras of interest, such as $\mathfrak{sl}(\infty)$, $\mathfrak{o}(\infty)$ and $\mathfrak{sp}(\infty)$). These are analogues of a result of Mürger's for compact groups and a number of results by the author on locally compact groups, and provide further evidence for the pervasiveness of such center-reconstruction phenomena.

Keywords: Lie algebra, primitive ideal, enveloping algebra, central character, induced representation, solvable, nilpotent, semisimple, Hopf algebra.

MSC: 17B05, 17B10, 16D60, 16T05.