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On the Universal L_∞-Algebroid of Linear Foliations

We compute an L_∞-algebroid structure on a projective resolution of some classes of singular foliations on a vector space V induced by the linear action of some Lie subalgebras of $\mathfrak{gl}(V)$. This L_∞-algebroid provides invariants of the singular foliations, and also provides a constant-rank replacement of the singular foliation. The computation consists of first constructing a projective resolution of the foliation induced by the linear action of the Lie subalgebra $\mathfrak{g} \subset \mathfrak{gl}(V)$, and then computing the L_∞-algebroid structure. We then generalize these constructions to a vector bundle E, where the role of the origin is now taken by the zero section L.
We then show that the fibers over a singular point of a projective resolution of any singular foliation can be computed directly from the foliation, without needing the projective resolution. For linear foliations, we also provide a way to compute the action of the isotropy Lie algebra in the origin on these fibers directly from the foliation.

Keywords: Singular foliations, L-infinity-algebroids, projective resolutions.

MSC: 22E45, 13D02, 17B55.