Representations of the Special Lie Superalgebra with $p$-Character of Height One

Let $k$ be an algebraically closed field of prime characteristic and $S(n)$ be the special Lie superalgebra of Cartan type over $k$. Define $\bar{S}(n) = S(n) \oplus k\xi_1D_1$. So $\overline{S}(n)_0 \cong \mathfrak{gl}(n)$. Let $\mathfrak{g} = S(n)$ or $\bar{S}(n)$. We investigate in this paper the representations of $\mathfrak{g}$ when $\chi$ is restricted or $\text{ht}(\chi) = 1$. The main results are listed below.

1. When $\text{ht}(\chi) = 1$, the irreducible representations of $U_\chi(\mathfrak{g})$ are considered. Precisely, the composition factors of the Kac modules are confirmed and the dimensions of simple modules are given.
2. When $\chi = 0$ or $\text{ht}(\chi) = 1$, the structures of indecomposable projective modules are studied and the Cartan invariants of $U_\chi(\mathfrak{g})$ are given.
3. When $\chi = 0$ or $\text{ht}(\chi) = 1$, we show that the representation category over $U_\chi(\mathfrak{g})$ has only one block (reckoning parities in).

Keywords: Special Lie superalgebra, irreducible representations, projective representations, Cartan invariants, block.

MSC: 17B10, 17B50, 17B35.