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Kronecker's Method and Complete Systems of Functions in Bi-Involution on Classical Lie Algebras

We use Kronecker's method to construct systems of functions in bi-involution with respect to two Poisson brackets: the canonical one and the bracket with frozen argument $A \in \mathfrak{g}$. For the Lie algebras \mathfrak{sl}_n and \mathfrak{sp}_{2n} , we construct complete systems of functions in bi-involution for any $A \in \mathfrak{g}$. For the Lie algebras \mathfrak{so}_{2n+1} and \mathfrak{so}_{2n} , we describe elements A such that we can construct a complete system of functions in bi-involution and the elements A such that we can construct the Kronecker part of a complete system of functions in bi-involution. Also, we prove that the constructed functions freely generate some limits of Mishchenko-Fomenko subalgebras. Finally, for the Lie algebras \mathfrak{sl}_n and \mathfrak{sp}_{2n} , we show that the Kronecker indices are the same for all elements A in any given sheet, while for the Lie algebras \mathfrak{so}_{2n} and \mathfrak{so}_{2n+1} , we give examples of sheets such that this is not true.

Keywords: Bi-Hamiltonian systems, Jordan-Kronecker invariants, argument shift method.

MSC: 17B80.