Journal of Lie Theory 33 (2023) 271–296

H. Glöckner
Institut für Mathematik, Universität Paderborn, Germany
glockner@math.upb.de

L. Tárrega
Departamento de Matemáticas, Universitat Jaume I, Castellón, Spain
ltarrega@uji.es

Mapping Groups Associated with Real-Valued Function Spaces and Direct Limits of Sobolev-Lie Groups

Let M be a compact smooth manifold of dimension m (without boundary) and G be a finite-dimensional Lie group, with Lie algebra $\mathfrak{g}$. Let $H^{>m/2}(M,G)$ be the group of all mappings $\gamma\colon M\to G$ which are H^s for some $s>\frac{m}{2}$. We show that $H^{>m/2}(M,G)$ can be made a regular Lie group in Milnor's sense, modelled on the Silva space $H^{>m/2}(M,\mathfrak{g}):=\lim\limits_{\longrightarrow\, s>m/2} H^s(M,\mathfrak{g})$, such that

$$H^{>m/2}(M,G) \;=\; \lim_{\longrightarrow\, s>m/2} H^s(M,G)$$

as a Lie group (where $H^s(M,G)$ is the Hilbert-Lie group of all G-valued H^s-mappings on M). We also explain how the (known) Lie group structure on $H^s(M,G)$ can be obtained as a special case of a general construction of Lie groups $\mathcal{F}(M,G)$ whenever function spaces $\mathcal{F}(U,\mathbb{R})$ on open subsets $U\subseteq\mathbb{R}^m$ are given, subject to simple axioms.

Keywords: Sobolev space, Banach space-valued section functor, mapping group, direct limit, pushforward, superposition operator, Nemytskij operator.

MSC: 22E65; 22E67, 46A13, 46E35, 46M40.