Stability of Geodesic Vectors in Low-Dimensional Lie Algebras

A naturally parameterised curve in a Lie group with a left invariant metric is a geodesic, if its tangent vector left-translated to the identity satisfies the Euler equation $\dot{Y} = Y Y$ on the Lie algebra of $G$. Stationary points (equilibria) of the Euler equation are called geodesic vectors: the geodesic starting at the identity in the direction of a geodesic vector is a one-parameter subgroup of $G$. We give a complete classification of Lyapunov stable and unstable geodesic vectors for metric Lie algebras of dimension 3 and for unimodular metric Lie algebras of dimension 4.

Keywords: Geodesic vector, Lie algebra, Lyapunov stability.