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Hardy Inequalities for Fractional (k, a)-Generalized Harmonic Oscillators

We define a-deformed Laguerre operators $L_{a,\alpha}$ and a-deformed Laguerre holomorphic semigroups on $L^2\left((0,\infty), d\mu_{a,\alpha}\right)$. Then we give a spherical harmonic expansion, which reduces to the Bochner-type identity when taking the boundary value $z = \pi i/2$, of the (k, a)-generalized Laguerre semigroup introduced by Ben Saïd, Kobayashi and Ørsted. We prove a Hardy inequality for fractional powers of the a-deformed Dunkl harmonic oscillator $\triangle_{k,a} := |x|^{2-a} \triangle_k - |x|^a$ using this expansion. When $a = 2$, the fractional Hardy inequality reduces to that of Dunkl-Hermite operators given by Ciaurri, Roncal and Thangavelu. The operators $L_{a,\alpha}$ also give a tangible characterization of the radial part of the (k, a)- generalized Laguerre semigroup on each k-spherical component $\mathcal{H}_k^m\left(\mathbb{R}^N\right)$ for

$$\lambda_{k,a,m} := \frac{2m + 2\langle k\rangle + N - 2}{a} \geq -\frac{1}{2}$$

defined via a decomposition of the unitary representation.

Keywords: Spherical harmonic expansion of (k,a)-generalized Laguerre semigroup, a-deformed Laguerre operators, fractional Hardy inequality, (k,a)-generalized harmonic oscillator.

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