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On Gradations of Decomposable Kac-Moody Lie Algebras by Kac-Moody Root Systems

We are interested in the gradations of symmetrizable Kac-Moody Lie algebras $\mathfrak{g}$ by root systems Σ of Kac-Moody type. We first show that we can reduce to the case where the grading root system Σ is indecomposable. If the graded Kac-Moody Lie algebra $\mathfrak{g}$ is decomposable, then any indecomposable component of $\mathfrak{g}$ is either fictive (and contributes little to the gradation) or effective (and essentially Σ-graded). Based on work by G. Rousseau and the first-named author, we extend most of the results on finite gradations to the gradations of $\mathfrak{g}$ admitting adapted root bases. Namely, it is shown that, for such a gradation, there exists a regular standard Kac-Moody-subalgebra $\mathfrak{g}(I_{re})$ of $\mathfrak{g}$ containing the grading Kac-Moody Lie subalgebra $\mathfrak{m}$ and which is finitely really Σ-graded. This enables us to investigate the structure of the Weyl group and the Tits cone of the grading Kac-Moody Lie subalgebra $\mathfrak{m}$ in comparison with those of the graded Kac-Moody Lie algebra $\mathfrak{g}$ and to prove a conjugacy theorem on adapted pairs of root bases. We end the paper by providing a unified construction for the finite imaginary gradations of $\mathfrak{g}$.

Keywords: Kac-Moody Lie algebra, gradation by a Kac-Moody root system, C-admissible pair.

MSC: 17B67.