Quantum Duality Principle for Quantum Continuous Kac-Moody Algebras

For the quantized universal enveloping algebra $U_h(\mathfrak{g}_X)$ associated with a continuous Kac-Moody algebra $\mathfrak{g}_X$ as in [A. Appel, F. Sala, Quantization of continuum Kac-Moody algebras, Pure Appl. Math. Q. 16 (2020), 439–493], we prove that a suitable formulation of the Quantum Duality Principle holds true, both in a "formal" version – i.e., applying to the original definition of $U_h(\mathfrak{g}_X)$ as a formal QUEA over $\mathbb{k}[[h]]$ – and in a "polynomial" one – i.e., for a suitable polynomial form of $U_h(\mathfrak{g}_X)$ over $\mathbb{k}[q, q^{-1}]$. In both cases, the QDP states that a suitable Hopf subalgebra of the given quantization of the Lie bialgebra $\mathfrak{g}_X$ is in fact a suitable quantization (in formal or in polynomial sense) of a connected Poisson group $G^*_X$ dual to $\mathfrak{g}_X$.

**Keywords:** Continuous Kac-Moody algebras, continuous quantum groups, quantization of Lie bialgebras, quantization of Poisson groups.

**MSC:** 17B37, 20G42; 17B65, 17B62.