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The Cortex of Nilpotent Lie Algebras of Dimensions Less or Equal to 7 and Semi-Direct Product of Vector Groups: Nilpotent Case

The paper deals with the cortex of real nilpotent Lie algebras. We first show that for any real nilpotent Lie algebra $\mathfrak{g}$ of dimension less or equal to 6, its cortex coincides with the set of the common zeros of the G-invariant polynomials on $\mathfrak{g}^\star$ namely the I-cortex, where G is the corresponding connected and simply connected Lie group and $\mathfrak{g}^\star$ is its dual. Next we give an example of 7-dimensional (real) nilpotent Lie algebra for which the cortex is a proper semi-algebraic set in the I-cortex. Finally we study the cortex of a class of nilpotent Lie groups given by a semi-direct product of abelian groups $G := \mathbb{R}^m \rtimes_\pi V$ where π is the continuous representation of $\mathbb{R}^n$ on the m-dimensional (real) vector space V defined by

$$\pi(t_1, \ldots, t_n) = \exp\left(\sum_{i=1}^{n} t_i A_i\right)$$

with $\{A_1, \ldots, A_n\}$ is a set of pairwise commuting nilpotent matrices in $\mathbb{R}^{m\times m}$.

Keywords: Nilpotent and solvable Lie groups, unitary representations of locally compact Lie groups.

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