On the Direct Integral Decomposition in Branching Laws for Real Reductive Groups

The restriction of an irreducible unitary representation $\pi$ of a real reductive group $G$ to a reductive subgroup $H$ decomposes into a direct integral of irreducible unitary representations $\tau$ of $H$ with multiplicities $m(\pi, \tau) \in \mathbb{N} \cup \{\infty\}$. We show that on the smooth vectors of $\pi$, the direct integral is pointwise defined. This implies that $m(\pi, \tau)$ is bounded above by the dimension of the space $\text{Hom}_H(\pi^\infty|_H, \tau^\infty)$ of intertwining operators between the smooth vectors, also called symmetry breaking operators, and provides a precise relation between these two concepts of multiplicity.

Keywords: Real reductive groups, unitary representations, branching laws, direct integral, pointwise defined, smooth vectors, symmetry breaking operators.

MSC: 22E45; 22E46.