

**S. O Rourke**

Dept. of Mathematics, Munster Technological University, Cork, Ireland  
shane.orourke1@mtu.ie

## **Affine Structures, Wreath Products and Free Affine Actions on Linear Non-Archimedean Trees**

Let  $\Lambda$  be an ordered abelian group,  $\text{Aut}^+(\Lambda)$  the group of order-preserving group automorphisms of  $\Lambda$ ,  $G$  a group and  $\alpha: G \rightarrow \text{Aut}^+(\Lambda)$  a homomorphism. An  $\alpha$ -affine action of  $G$  on a  $\Lambda$ -tree  $X$  is one that satisfies  $d(gx, gy) = \alpha_g d(x, y)$  ( $x, y \in X$ ,  $g \in G$ ). We consider classes of groups that admit a free affine action in the case where  $X = \Lambda$ . Such groups form a much larger class than in the isometric case. We show in particular that unitriangular groups  $\text{UT}(n, \mathbb{R})$  and groups  $T^*(n, \mathbb{R})$  of upper triangular matrices over  $\mathbb{R}$  with positive diagonal entries admit free affine actions. Our proofs involve left symmetric structures on the respective Lie algebras and the associated affine structures on the groups in question. We also show that given ordered abelian groups  $\Lambda_0$  and  $\Lambda_1$  and a free order-preserving affine action of  $G$  on  $\Lambda_0$ , we obtain another such action of the wreath product  $G \wr \Lambda_1$  on a suitable  $\Lambda'$ .

It follows that all free soluble groups, residually free groups and locally residually torsion-free nilpotent groups admit free affine actions on some  $\Lambda'$ .

**Keywords:** Group actions on  $\Lambda$ -trees, upper triangular groups, affine structures, wreath products.

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