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Quadratic Forms on the 27-Dimensional Modules for E_6 in Characteristic Two

The purpose of this paper is to study the Chevalley group E of type $E_6(\mathbb{K})$ over fields $\mathbb{K}$ of characteristic two. We use the generalized quadrangle $(\mathbb{P},)$ over $\mathbb{K}$ of type $O_6^-(2)$ to construct a trilinear form T on a 27-dimensional vector space A, this form preserves the action of E. We introduce an involution

$$g \to g^\alpha = g^* = (g^t)^{-1}$$

on E, algebra structure on A and a quadratic map $\hat{Q} : A \to A$. Then we prove the following results:

(a) $\hat{Q}(x^g) = \hat{Q}(x)^{g^*}$ for all $x \in A$ and $g \in E$.

(b) For $x, y, z \in A$ and $g \in E$, the following holds true:
(1) $x^g\, y^g = (xy)^{g^*}$,
(2) $T(x^g, y^g, z^g) = T(x, y, z)$.

(c) The main results:
(1) The group G of isometries of T coincides with the group $G^* = \{g \in GL(A) \mid a^g b^g = (ab)^{g^*}\}$.
(2) The group $G_0 = \{g \in GL(A) \mid \hat{Q}(a^g) = \hat{Q}(a)^{g^*}\}$ is intermediate between E and G.
(3) The group $E = E^* = \{g^* = (g^t)^{-1} \mid g \in E\}$.

Keywords: Quadratic forms, generalized quadrangles, groups of Lie type.

MSC: 17A75, 17A45.