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Nets of Standard Subspaces Induced by Antiunitary Representations of Admissible Lie Groups I

Let $(\pi, \mathcal{H})$ be a strongly continuous unitary representation of a 1-connected Lie group G such that the Lie algebra $\mathfrak{g}$ of G is generated by the positive cone $C_\pi := \{x \in \mathfrak{g} : -i\partial\pi(x) \geq 0\}$ and an element h for which the adjoint representation of h induces a 3-grading of $\mathfrak{g}$. Moreover, suppose that $(\pi, \mathcal{H})$ extends to an antiunitary representation of the extended Lie group $G_\tau := G \rtimes \{\mathbf{1}, \tau_G\}$, where τ_G is an involutive automorphism of G with $\mathbf{L}(\tau_G) = e^{i\pi \, \mathrm{ad}\, h}$. In a recent work by Neeb and Ólafsson, a method for constructing nets of standard subspaces of $\mathcal{H}$ indexed by open regions of G has been introduced and applied in the case where G is semisimple. In this paper, we extend this construction to general Lie groups G, provided the above assumptions are satisfied and the center of the ideal $\mathfrak{g}_C = C_\pi - C_\pi \subset \mathfrak{g}$ is one-dimensional. The case where the center of $\mathfrak{g}_C$ has more than one dimension will be discussed in a separate paper.

Keywords: Standard subspace, Lie group, covariant net, quantum field theory.

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