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Vertices of Intersection Polytopes and Rays of Generalized Kostka Cones

Let $\mathcal{K}(G)$ be the rational cone generated by pairs (λ, μ) where λ and μ are dominant integral weights and μ is a nontrivial weight space in the representation V_λ of a semisimple group G. We produce all extremal rays of $\mathcal{K}(G)$ by considering the vertices of corresponding intersection polytopes IP_λ, the set of points in $\mathcal{K}(G)$ with first coordinate λ. We show that vertices of IP_{ϖ_i} arise as lifts of vertices coming from cones $\mathcal{K}(L)$ associated to simple Levi subgroups possessing the simple root α_i. As corollaries we obtain a complete description of all extremal rays, as well as polynomial formulas describing the numbers of extremal rays depending on type and rank.

Keywords: Representation theory, convex geometry, Lie combinatorics, Kostka numbers, weight polytopes.

MSC: 22E46, 05E10, 52A40.