Homological Finiteness of Representations of Almost Linear Nash Groups

Let $G$ be an almost linear Nash group, namely, a Nash group that admits a Nash homomorphism with finite kernel to some $GL_k(\mathbb{R})$. A smooth Fréchet representation $V$ with moderate growth of $G$ is called homologically finite if the Schwartz homology $H^i_S(G; V)$ is finite dimensional for every $i \in \mathbb{Z}$. We show that the space of Schwartz sections $\Gamma^\psi(X, E)$ of a tempered $G$-vector bundle $(X, E)$ is homologically finite as a representation of $G$, under some mild assumptions.

**Keywords**: Schwartz homology, tempered vector bundle, Schwartz sections, homological finiteness.

**MSC**: 22E41.