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## Structure and Representations for the Electrical Lie Algebra of Type $D_4$

We prove the dimension conjecture for electrical Lie algebra  $\mathfrak{e}_{D_4}$  of type  $D_4$ . Moreover, we present a new method to construct 3-step nilpotent Lie algebras and show that  $\mathfrak{e}_{D_4}$  is isomorphic to the semidirect product of  $\mathfrak{sl}_2$  with a 3-step nilpotent Lie algebra constructed from the colored complete bipartible graph  $K_{2,2}$ . Also, we classify all simple highest weight modules for  $\mathfrak{e}_{D_4}$ .

**Keywords**: Electrical Lie algebras, 3-step nilpotent Lie algebra, highest weight modules, simple modules.

MSC: 17B10, 17B20, 17B65, 17B66, 17B68.