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Transitive Lie Algebras of Nilpotent Vector Fields and their Tanaka Prolongations

Transitive nilpotent local Lie algebras of vector fields can be easily constructed from dilations h of $\mathbb{R}^n$ with positive weights (give me a sequence of n positive integers and I will give you a transitive nilpotent Lie algebra of vector fields on $\mathbb{R}^n$) as the Lie algebras $\mathfrak{g}_{<0}(h)$ of the polynomial vector fields of negative weights with respect to h.
We provide a condition for the dilation h such that the Lie algebras of polynomial vectors defined by h are exactly the Tanaka prolongations of the corresponding nilpotent Lie algebras $\mathfrak{g}_{<0}(h)$. However, in some cases of dilations h we can find some 'strange' elements of the Tanaka prolongations of $\mathfrak{g}_{<0}(h)$, which we describe in detail. In particular, we give a complete description of derivations of degree 0 for the Lie algebra $\mathfrak{g}_{<0}(h)$.

Keywords: Vector field, nilpotent Lie algebra, dilation, derivation, homogeneity structures.

MSC: 17B30, 17B66; 57R25, 57S20.