On Epimorphisms in some Categories of Infinite-Dimensional Lie Groups

Let $X$ be a smooth compact connected manifold. Let $G = \text{Diff} X$ be the group of diffeomorphisms of $X$, equipped with the $C^\infty$-topology, and let $H$ be the stabilizer of some point in $X$. Then the inclusion $H \to G$, which is a morphism of two regular Fréchet-Lie groups, is an epimorphism in the category of smooth Lie groups modelled on complete locally convex spaces. At the same time, in the latter category, epimorphisms between finite dimensional Lie groups have dense range. We also prove that if $G$ is a Banach-Lie group and $H$ is a proper closed subgroup, the inclusion $H \to G$ is not an epimorphism in the category of Hausdorff topological groups.

Keywords: Epimorphism, locally convex Lie group, Frechet-Lie group, Banach-Lie group, Hausdorff topological group.

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