Journal of Lie Theory 31 (2021) 719–750

A. Baklouti
Dept. of Mathematics, Faculty of Science of Sfax, Sfax, Tunisia
ali.baklouti@fss.usf.tn

A. Sasaki
Dept. of Mathematics, Tokai University, Kanagawa, Japan
and: Dept. of Mathematics, Friedrich-Alexander-Universität, Erlangen, Germany
atsumu@tokai-u.jp

Visible Actions and Criteria for Multiplicity-Freeness of Representations of Heisenberg Groups

A visible action on a complex manifold is a holomorphic action that admits a J-transversal totally real submanifold S. It is said to be strongly visible if there exists an orbit-preserving anti-holomorphic diffeomorphism σ such that $\sigma|_S = \mathrm{id}_S$. Let G be the Heisenberg group and H a non-trivial connected closed subgroup of G. We prove that any complex homogeneous space $D = G^{\mathbb{C}}/H^{\mathbb{C}}$ admits a strongly visible L-action, where L stands for a connected closed subgroup of G explicitly constructed through a co-exponential basis of H in G. This leads in turn that G itself acts strongly visibly on D. The proof is carried out by finding explicitly an orbit-preserving anti-holomorphic diffeomorphism and a totally real submanifold S, for which the dimension depends upon the dimensions of G and H. As a direct application, our geometric results provide a proof of various multiplicity-free theorems on continuous representations on the space of holomorphic sections on D. Moreover, we also generate as a consequence, a geometric criterion for a quasi-regular representation of G to be multiplicity-free.

Keywords: Visible action, slice, Heisenberg group, Heisenberg homogeneous space, multiplicity-free representation.

MSC: 22E25; 22E27.