Affine Schur Duality

The Schur duality may be viewed as the study of the commuting actions of the symmetric group $S_d$ and the general linear group $GL(n, \mathbb{C})$ on $E \otimes^d$ where $E = \mathbb{C}^n$. Here we extend this duality to the context of the affine Weyl (or symmetric) group $\mathbb{Z}^d \rtimes S_d$ and the affine Lie (or Kac-Moody) algebra $\mathfrak{g} = \mathcal{L}g \oplus \mathbb{C}c$, $\mathfrak{g} = \mathfrak{sl}_n(\mathbb{C})$. Thus we construct a functor $\mathcal{F} : M \mapsto M \otimes_{S_d} E \otimes^d$ from the category of finite dimensional $\mathbb{C}[\mathbb{Z}^d \rtimes S_d]$-modules $M$ to that of finite dimensional $\mathfrak{g}$-modules $W$ of level 0 (the center $\mathbb{C}c$ of $\mathfrak{g}$ acts as zero, thus these are representations of the loop group $\mathcal{L}g = \mathcal{L} \otimes_{\mathbb{C}} \mathfrak{g}$, where $\mathcal{L} = \mathbb{C}[t, t^{-1}]$, $\mathfrak{g} = \mathfrak{sl}_n(\mathbb{C})$), the irreducible constituents of whose restriction to $\mathfrak{g}$ are subrepresentations of $E \otimes^d$.

When $d < n$ it is an equivalence of categories, but not for $d = n$, in contrast to the classical case. As an application we conclude that all irreducible finite dimensional representations of $\mathcal{L}g$, the irreducible constituents of whose restriction to $\mathfrak{g}$ are subquotients of $E \otimes^d$, are tensor products of evaluation representations at distinct points of $\mathbb{C}^\times$.

Keywords: Affine Schur duality, affine Lie algebra, affine Kac-Moody algebra, loop group, loop algebra, affine Lie group, evaluation representations, finite dimensional representations.