Journal of Lie Theory 31 (2021) 681–718

Y. Z. Flicker
Ariel University, Ariel 40700, Israel
and: The Ohio State University, Columbus, OH 43210-1174, U.S.A.
yzflicker@gmail.com

Affine Schur Duality

The Schur duality may be viewed as the study of the commuting actions of the symmetric group S_d and the general linear group $\mathrm{GL}(n,\mathbb{C})$ on $\mathbb{E}^{\otimes d}$ where $\mathbb{E}=\mathbb{C}^n$. Here we extend this duality to the context of the affine Weyl (or symmetric) group $\mathbb{Z}^d \rtimes S_d$ and the affine Lie (or Kac-Moody) algebra $\widetilde{\mathfrak{g}}=\mathcal{L}\mathfrak{g}\oplus\mathbb{C}c$, $\mathfrak{g}=\mathrm{sl}_n(\mathbb{C})$. Thus we construct a functor $\mathcal{F}: M \mapsto M \otimes_{S_d} \mathbb{E}^{\otimes d}$ from the category of finite dimensional $\mathbb{C}[\mathbb{Z}^d \rtimes S_d]$-modules M to that of finite dimensional $\widetilde{\mathfrak{g}}$-modules W of level 0 (the center $\mathbb{C}c$ of $\widetilde{\mathfrak{g}}$ acts as zero, thus these are representations of the loop group $\mathcal{L}\mathfrak{g}=\mathcal{L}\otimes_{\mathbb{C}}\mathfrak{g}$, where $\mathcal{L}=\mathbb{C}[t,t^{-1}]$, $\mathfrak{g}=\mathrm{sl}_n(\mathbb{C})$), the irreducible constituents of whose restriction to $\mathfrak{g}$ are subrepresentations of $\mathbb{E}^{\otimes d}$. When $d<n$ it is an equivalence of categories, but not for $d=n$, in contrast to the classical case. As an application we conclude that all irreducible finite dimensional representations of $\mathcal{L}\mathfrak{g}$, the irreducible constituents of whose restriction to $\mathfrak{g}$ are subquotients of $\mathbb{E}^{\otimes d}$, are tensor products of evaluation representations at distinct points of $\mathbb{C}^\times$.

Keywords: Affine Schur duality, affine Lie algebra, affine Kac-Moody algebra, loop group, loop algebra, affine Lie group, evaluation representations, finite dimensional representations.

MSC: 17B10, 17B20, 17B65, 17B67, 22E50, 22E65, 22E67.