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F. Naef
School of Mathematics, Trinity College, Dublin, Ireland
naeff@tcd.ie

Y. Qin
Massachusetts Institute of Technology, Cambridge, U.S.A.
emmaqin@mit.edu

The Elliptic Kashiwara-Vergne Lie Algebra in Low Weights

We study the elliptic Kashiwara-Vergne Lie algebra $\mathfrak{krv}$, which is a certain Lie subalgebra of the Lie algebra of derivations of the free Lie algebra in two generators. It has a natural bigrading, such that the Lie bracket is of bidegree $(-1,-1)$. After recalling the graphical interpretation of this Lie algebra, we examine low degree elements of $\mathfrak{krv}$. More precisely, we find that $\mathfrak{krv}^{(2,j)}$ is one-dimensional for even j and zero for j odd. We also compute

$$\dim(\mathfrak{krv})^{(3,j)} = \lfloor\frac{j-1}{2}\rfloor - \lfloor\frac{j-1}{3}\rfloor.$$

In particular, we show that in those degrees there are no odd elements and also confirm Enriquez' conjecture in those degrees.

Keywords: Elliptic Kashiwara-Vergne Lie algebra.

MSC: 17B01