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The Liouville Theorem of a Torsion System and its Application to the Symmetry Group of a Porous Medium Type Equation on Symmetric Spaces

We first prove a Liouville theorem to the torsion system

$$\begin{cases} \xi_i^i = \lambda(x) \pm \frac{2x^k \xi^k}{|x|^2 + 1}, & \forall i = 1, 2, \dots, n \\ \xi_j^i + \xi_i^j = 0, & \forall i \neq j \end{cases}$$

for $(\xi, \lambda) \in C^{\infty}(\mathbb{R}^n, \mathbb{R}^n \times \mathbb{R})$. As an application, complete resolutions of symmetry groups to the porous medium equation

$$u_t - \triangle_q(u^m) = u^p, \ \forall (x,t) \in M \times \mathbb{R}$$

of Fujita type are obtained, where M is the sphere $\mathbb{S}^n \subset \mathbb{R}^{n+1}$ or hyperbolic space \mathbb{H}^n with canonical metric g.

Keywords: Porous medium equation, prolongation formula.

MSC: 53C35, 35K59, 35K65.