The Liouville Theorem of a Torsion System and its Application to the Symmetry Group of a Porous Medium Type Equation on Symmetric Spaces

We first prove a Liouville theorem to the torsion system

\[
\begin{cases}
\xi_i = \lambda(x) + \frac{2x^k\xi_k}{|x|^2 + 1}, & \forall i = 1, 2, \cdots, n \\
\xi_i + \xi_j = 0, & \forall i \neq j
\end{cases}
\]

for \((\xi, \lambda) \in C^\infty(\mathbb{R}^n, \mathbb{R}^n \times \mathbb{R})\). As an application, complete resolutions of symmetry groups to the porous medium equation

\[u_t - \Delta_g(u^m) = u^p, \quad \forall (x, t) \in M \times \mathbb{R}\]

of Fujita type are obtained, where \(M\) is the sphere \(\mathbb{S}^n \subset \mathbb{R}^{n+1}\) or hyperbolic space \(\mathbb{H}^n\) with canonical metric \(g\).

**Keywords:** Porous medium equation, prolongation formula.

**MSC:** 53C35, 35K59, 35K65.