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Universal Averages in Gauge Actions

We give a construction of a universal average of Lie algebra elements whose exponentiation gives (when there is an associated Lie group) a totally symmetric geometric mean of Lie group elements (sufficiently close to the identity) with the property that in an action of the group on a space X for which n elements all take a particular point $a \in X$ to a common point $b \in X$, also the mean will take a to b. The construction holds without the necessity for the existence of a Lie group and the universal average $\mu_n(x_1, \ldots, x_n)$ is a totally symmetric universal expression in the free Lie algebra generated by $x_1, \ldots, x_n$. Its expansion up to three brackets is found explicitly and various properties of iterated averages are given. Although this is a purely algebraic result, it is expected to have applications in diverse fields. One known application is to the construction of explicit differential graded Lie algebra models of three dimensional cells and thereby to discretised differential geometry on cubulated manifolds. This work is based on the second author's minor thesis.

Keywords: DGLA, Maurer-Cartan, Baker-Campbell-Hausdorff formula, Karcher mean.

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