Spinorial Representations of Orthogonal Groups

Let $G$ be a real compact Lie group, such that $G = G^0 \rtimes C_2$, with $G^0$ simple. Here $G^0$ is the connected component of $G$ containing the identity and $C_2$ is the cyclic group of order 2. We give criteria for whether an orthogonal representation $\pi: G \to O(V)$ lifts to $\text{Pin}(V)$ in terms of the highest weights of $\pi$ and also in terms of character values. From these criteria we compute the first and second Stiefel-Whitney classes of the representations of the orthogonal groups.

**Keywords:** Orthogonal group, spinorial representation, Stiefel-Whitney class, highest weight.

**MSC:** 22E41, 22E47, 57R20.